Mathematical Techniques for Advanced Economic Analysis (MTAEA)
School of Economics, The Australian National University
January 11-February 19, 2010

Updated: 25 Nov 2009

Instructor: Scott McCracken

Phone: 6125 6168

Email: scott.mccracken@anu.edu.au

Office Hours: By appointment in HW Arndt 2033.

Description: This *fee-free* course comprises six weeks of intensive instruction and assessment commencing 11 January 2010. It is intended for students enrolling in: (i) Part A of the PhD in Economics; (ii) the Masters program in Economics; or (iii) other Economics programs by permission of the instructor.

The aim of this course is to provide students with the basic mathematical tools required for higher level study in economics, and particularly the courses Microeconomic Theory A and Macroeconomic Theory. We will cover the fundamental concepts of set theory, matrices, vectors, sequences, limits and differentiation, as well as the essentials of static optimization.

Enrolment: There is no formal enrolment for this course – instead, email the lecturer to inform that you will sit the course. In the email include your full name and student ID, the degree you will pursue in 2010, and briefly explain your mathematical background.

Lectures and Tutorials: In the six weeks from 11 January to 19 February we will meet as follows in Arndt Tutorial Room 7: Mondays and Fridays (4-6pm) and Tuesdays and Thursdays (9-11am). Since Tuesday 26 January is Australia Day, there will be a lecture on Wednesday (4-6pm) that week. Notes and other material can be found here.

Bedtime Reading: The core textbook will be Simon and Blume (1994) and Chapters 1-5 are *required preliminary reading*. It is available at the Co-op Bookshop on campus, although you might consider purchasing it online at amazon.com or elsewhere. This and other references used in the course are listed at the end of the document.

(Tentative) Course Outline:

Week 1

- **Set Theory** (Bloch ch. 3)
  - Basic definitions, examples (integers, real numbers, intervals, power sets), notation, set operations, properties (examples of proofs).
- **Relations** (Bloch ch. 5, Ok A.1.2-A.1.4)
  - Cartesian products, relations, properties of relations, equivalence relations and partitions, order relations.
- **Functions & Cardinality** (Bloch chs 4,6)
  - Definition of a function, image, inverse image, composition, finite and infinite sets, countability, examples using the number system, induction.

Week 2

- **Matrices & Systems of Linear Equations** (S&B chs. 7-8)
  - Definition, matrix algebra, inverses, solutions of linear systems.
- **Determinants & More Linear Systems** (S&B ch. 9)
  - Definitions, invertible matrices and their determinants, conditions for solvability of linear systems.
- **Vectors in Euclidean Spaces** (S&B ch. 10)
  - Vector algebra, inner products, projections.

Week 3

- **Linear Independence** (S&B ch. 11)
  - Definition, conditions for linear independence using determinant, generating sets, basis and dimension.
- **Sequences on the Real Line** (Ross 2.7-2.11)
  - Convergence, limits, infima and suprema, subsequences, monotone sequences, Cauchy sequences.
- **Continuity and Limits of Functions** (Ross 3.17-3.18, 3.20, S&B p22-34)
  - Definition, properties, intermediate value theorem, limits of functions.

Week 4

- **Differentiation** (Ross 3.28-3.29, S&B p22-34)
  - Definition and properties, chain rule, mean value theorem, Taylor series.
- **Multivariable Calculus** (S&B chs 13-14)
  - Functions of several variables, level curves, partial derivatives, total derivative.
- **Directional Derivatives and Gradients** (S&B chs 13-14)
  - Directional derivatives, linear transformations, implicit function theorem.

Week 5

- **Metric Space Properties of n-dimensional Euclidean Space** (S&B ch.12)
  - Sequences, convergence, open, closed, and compact sets.
- **Optimization in Euclidean Space**
  - Local and global optima, first and second order conditions, Weierstrass theorem.
- **Equality Constraints and Lagrangians**
  - Lagrangians and multipliers, Hessians and second order conditions.

Week 6

- **Inequality-Constrained Optimization**
  - The Kuhn-Tucker theorem, multipliers and constraint qualifications, envelope theorem.
- **Convexity and Quasiconvexity**
  - Convexity of sets and functions, properties, implications for optimization problems.
- **Correspondences**
  - Upper and lower hemicontinuity, the maximum theorem.

References: Textbooks used in constructing the course and which you may find useful are listed below – most are available in the university library. Note that some are at a more advanced level. The books in bold are on 2 hour reserve in Chifley Library.

1. **Bloch, Ethan D. (2000). *Proofs and Fundamentals: A First Course in Abstract Mathematics*, Birkhaeuser.**

2. Ok, Efe A. (2007). *Real Analysis with Economic Applications*, Princeton.*

3. Ross, Kenneth A. (1980). *Elementary Analysis: The Theory of Calculus*, Springer.†

4. **Simon, Carl P. and Lawrence Blume (1994). *Mathematics for Economists*, WW Norton.**

5. **Sundaram, Rangarajan K. (1996). *A First Course in Optimization Theory*, Cambridge.**

6. Sydsaeter, Knut and Peter Hammond (2006). *Essential Mathematics for Economic Analysis*, Prentice Hall.

*Ch. A available online at `http://press.princeton.edu/chapters/s8274.pdf`.

†Ch. 2 available online at `http://anulib.anu.edu.au/anuonly/ereserve/ssh/2007/sem2/22997787.pdf`.