Implicit Functions Exercises

All the exercises are from Simon & Blume.

1. Exercises 15.1-15.4.

2. Exercise 15.6.

3. Exercise 15.9.

4. Exercises 15.17-15.19.

5. Exercise 15.22.

6. Exercises 15.24-15.25

7. Some economics! Consider a closed economy with an imperfectly competitive industry producing a good under increasing returns to scale.

   Let Q denote the total demand for this good. Denoting utility by $U = U(Q)$ (which is assumed to be three times differentiable) and denoting the inverse demand function by $p = p(Q)$, we have $p(Q) = U'(Q)$ and $p'(Q) = U''(Q) < 0$.

   There are assumed to be n identical firms and we denote a single representative firm's sales by y. Since the quantity sold must be equal to the quantity produced we have the following market-clearing condition:

   $$Q = ny. \tag{1}$$

   Each firm faces a fixed cost f, a constant marginal cost of c, and a given specific tax per unit of t. The profits of a representative firm are thus given by $\Pi = (p - c - t)y - f$, leading to the first order condition for profit maximization

   $$\Pi_y = (p - c - t) + p'y = 0. \tag{2}$$

   Here we assume Cournot competition so that each firm takes as given the output of the other firms when choosing its own output. Furthermore, we assume the second order condition for profit maximization holds, i.e. $\Pi_{yy} = 2p'(z) + p''(z)y < 0$, for all $z \in [0, Z]$, where Z is the aggregate output which would result in zero profits. With free entry and exit a representative firm's profits are driven to zero, i.e.

$$\Pi = (p - c - t)y - f = 0. \tag{3}$$

(a) Partition t, y, n and Q into endogenous and exogenous variables.

(b) Assume that the inverse demand function is linear i.e. $p(Q) = 1 - Q$. Solve the resulting system explicitly for the endogenous variables as functions of the exogenous variable(s) and calculate the effects of a change in the tax rate.

(c) Now assume demand may be nonlinear. Use the implicit function theorem and equations (1) to (3) to find the effect of a change in the exogenous variable(s) on the endogenous variables. Find the sign of these changes when inverse demand is convex ($p'' > 0$), when it is concave ($p'' < 0$) and when it is linear ($p'' = 0$).

(d) Now suppose the government sets the tax t so as to maximize social welfare which is the sum of consumer surplus and tax revenue, *i.e.*,

$$W = U(Q) - p(Q)Q + tQ.$$

Suppressing dependence on Q, the effect on welfare of a change in the tax rate is then given by

$$\begin{aligned}\frac{dW}{dt} &= U'\frac{dQ}{dt} - p'\frac{dQ}{dt}Q - p\frac{dQ}{dt} + Q + t\frac{dQ}{dt} \\ &= (t - p'Q)\frac{dQ}{dt} + Q.\end{aligned}$$

Find the optimal tax rate using the first order condition (assuming the second order conditions for a welfare maximum hold) and sign it for different values of p''.