

If you wish to discuss the tutorial questions please come to my [office hours](#).

1. Consider Hotelling's model of product differentiation (with a uniform distribution of consumers). Suppose all firms are required to charge the same fixed price (which is above the constant marginal cost). Given this assumption, each consumer will purchase from the firm closest to them (if a group of consumers is equally close to multiple firms, then those firms share the demand from that group equally).
 - (a) Explain why, when there are three firms, there is no Nash equilibrium in locations.¹
[Hint: Consider three possibilities: (i) the three firms locate in different positions; (ii) two firms locate in the same position, the other in a different position; (iii) the firms all locate in the same position. Show there is a profitable deviation in each case.]
 - (b) Find the Nash equilibrium when there are four firms. Explain why it is a Nash equilibrium. What are the efficient locations (i.e. the locations that minimize total 'distance' travelled by consumers)?
2. Despite low barriers to entry into the US ready-to-eat breakfast cereal industry, there was virtually no entry of new firms into this industry between the 1950s and 1970s even though incumbent firms (Kellogg, General Mills, General Foods, and Quaker Oats) made significant profits. During the same period, the number of brands sold by those incumbent firms increased from 25 to about 80. In order to explain this observations, we will consider a simple model.

An incumbent monopoly in the cereal industry is facing potential entry from a rival. In an attempt to deter entry, the incumbent is considering whether to increase its offering from one variety of cereal to two varieties of cereal. We will investigate how this can be a worthwhile strategy.

In order to keep things relatively simple, suppose that the price of all varieties of cereal is fixed at 100. The fixed cost of producing each variety is F where $0 < F < 50$. These assumptions mean that the profit of a firm is just $100s - nF$, where s is its market share and n is the number of varieties it produces. A choice of variety is a choice of location on the Hotelling line between 0 and 1. As in the standard Hotelling model, consumers are uniformly distributed along the line. Consumers will purchase the variety of the good that is closest to them.

Consider the following two stage game:

Stage 1: Incumbent chooses number of varieties (1 or 2) and location(s)
Stage 2: Rival chooses number of varieties (0 or 1) and location

A simplified game tree is illustrated below.²

¹More precisely, you need to explain why there is no *pure* strategy Nash equilibrium. There *is* actually a Nash equilibrium, but it involves *mixed* strategies (the firms randomize their locations).

²The game tree drawn is not the full game tree as it does not have any decision nodes for the firms' choices of variety. It is instead drawn given the two firms' equilibrium choices of variety.

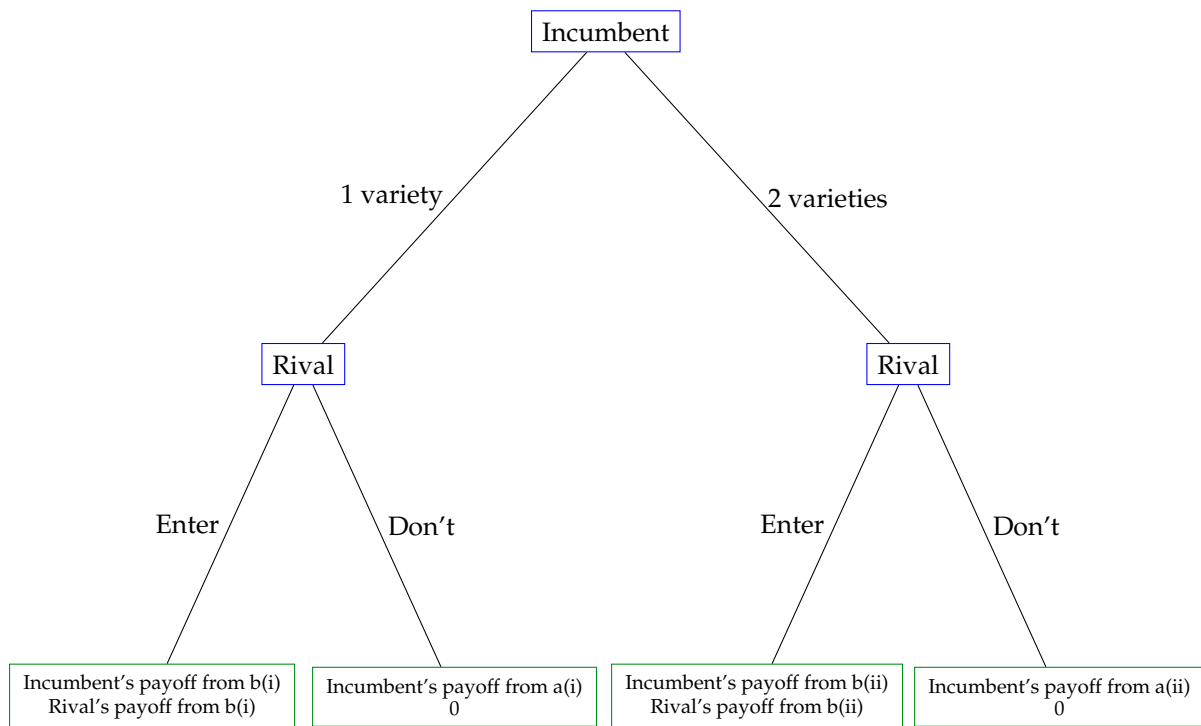


Figure 1: Product proliferation game (given equilibrium locations)

Redraw the above game tree and then fill in the payoffs in the last row of boxes as you work through parts (a) and (b) below. The resulting diagram should help you answer (c).

- (a) Suppose that the potential rival decides not to enter in the second stage (i.e. produces zero varieties).
 - (i) What is the incumbent's profit if it chooses to produce one variety?
 - (ii) What is the profit if it chooses to produce two varieties?
 - (iii) Would the incumbent wish to produce one variety or two varieties if there were no threat of entry?
- (b) Now suppose that the rival *does* decide to enter in the second stage and produce one variety. Given any first-stage location choice(s) of the incumbent, the rival will choose the location of its variety to maximize its profit.
 - (i) Suppose the incumbent produces only one variety. Where should the incumbent locate its variety to maximize its profit? What is the rival's location? What are the incumbent's and rival's profits? Will the rival actually wish to enter?
 - (ii) Suppose the incumbent produces two varieties. Where should the incumbent locate its varieties to maximize its profit? What is the rival's location? What are the incumbent's and rival's profits? For what range of F will the rival not wish to enter?
- (c) For what range of F is it worthwhile for the incumbent to deter entry by producing two varieties instead of one?